Exercise 3.5C A1 on CD

Sketch the following circles. (Remember to show coordinates of centres and and axis intercepts clearly.)

1 $x^2 + y^2 = 25$

2 $(x + 3)^2 + (y - 2)^2 = 4$

3 $(x + 1)^2 + (y - 6)^2 = 49$

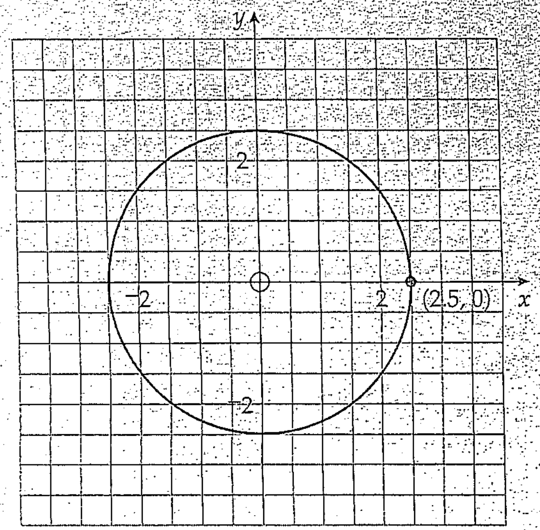
4 $x^2 + (y - 4)^2 = 16$

5 $x^2 - 10x + y^2 - 8y = 8$ (You will need to complete the square first.)

6 $x^2 - 4x + y^2 - 4y + 7 = 0$

Find the equations of the following graphs:

7



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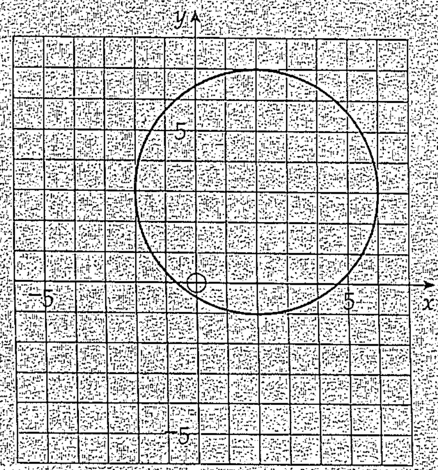

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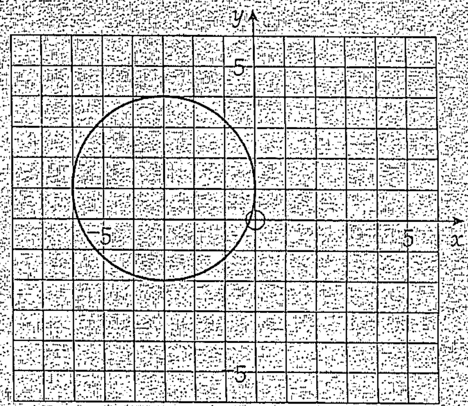

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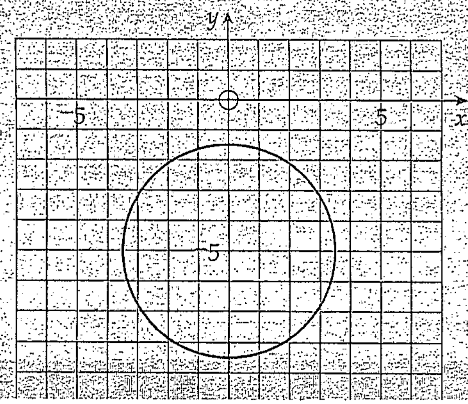


10 Conic Sections — Achieved

Exercise 3.5C A1

1 Centre (0, 0), radius 5 units

2 Centre (⁻3, 2), radius 2 units

3 Centre (⁻1, 6), radius 7 units

4 Centre (0, 4), radius 4 units

5 Centre (5, 4), radius 7 units

6 Centre (2, 2), radius 1 unit

7 Equation: $x^2 + y^2 = 6.25$

8 Equation: $(x - 2)^2 + (y - 3)^2 = 16$

9 Equation: $(x + 3)^2 + (y - 1)^2 = 9$

10 Equation: $x^2 + (y + 5)^2 = 12.25$

Exercise 3.5C A2 on CD

Sketch the following ellipses. (Remember to show coordinates of centres and axis intercepts clearly.)

1 $\frac{x^2}{9} + \frac{y^2}{4} = 1$

2 $\frac{(x-1)^2}{16} + \frac{(y+2)^2}{9} = 1$

3 $\frac{(x+1)^2}{36} + \frac{(y-3)^2}{4} = 1$

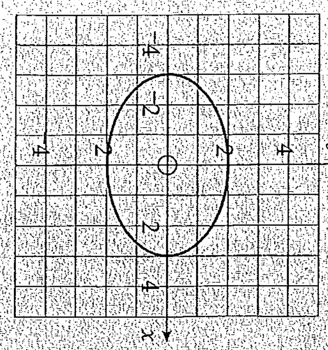
4 $x^2 + \frac{(y-3)^2}{9} = 4$

5 $9x^2 + 4y^2 = 36$

6 $4x^2 + 16x + y^2 - 2y + 13 = 0$ (You will need to complete the square first.)

Find the equations of the following ellipses.

7



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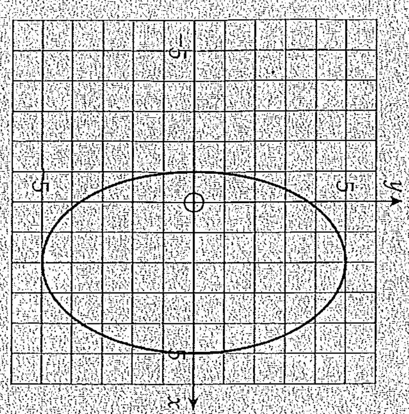

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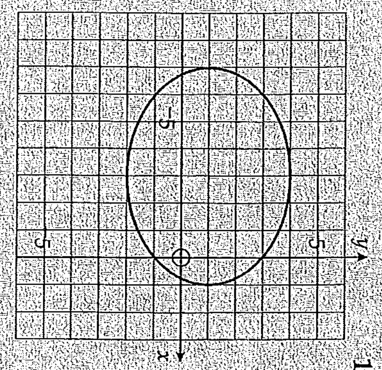

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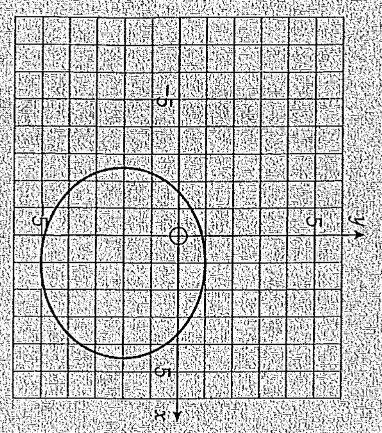


Exercise 3.5C A2

1 Centre (0, 0), semi-major axis length 3, semi-minor axis length 2

2 Centre (1, ⁻2), semi-major axis length 4, semi-minor axis length 3

3 Centre (⁻1, 3), semi-major axis length 6, semi-minor axis length 2

4 Centre (0, 3), horizontal semi-axis length 2, vertical semi-axis length 6

5 Centre (0, 0), horizontal semi-axis length 2, vertical semi-axis length 3

6 Centre (⁻2, 1), horizontal semi-axis length 1, vertical semi-axis length 2

7 Equation: $\frac{x^2}{9} + \frac{y^2}{4} = 1$

8 Equation: $\frac{(x-2)^2}{9} + \frac{y^2}{25} = 1$

9 Equation: $\frac{(x+3)^2}{16} + \frac{(y-1)^2}{9} = 1$

10 Equation: $\frac{(x-1)^2}{12.25} + \frac{(y+2)^2}{9} = 1$

CD **Exercise 3.5C A3 on CD**

Sketch the following parabolas. (Remember to show coordinates of vertices and axis intercepts clearly.)

1 $y^2 = 12x$ 2 $y^2 = 25x$

3 $(y-2)^2 = 8(x+3)$ 4 $(y-1)^2 = 12x$

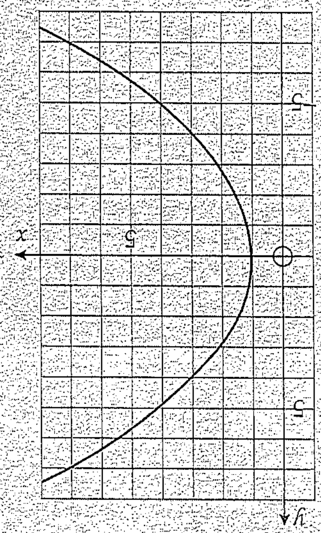
5 $y^2 = 6x$

(continued)

6 $(x-1)^2 = 12(y+2)$ (Notice this one is arranged vertically.)

Find the equations of the following parabolae:

7



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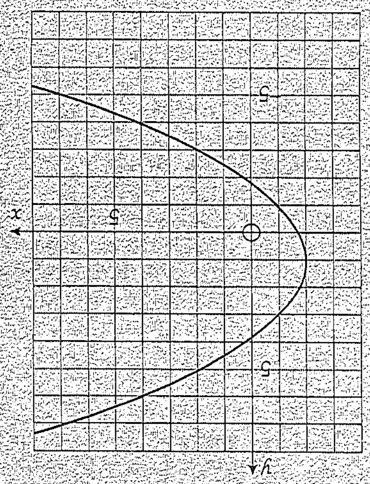

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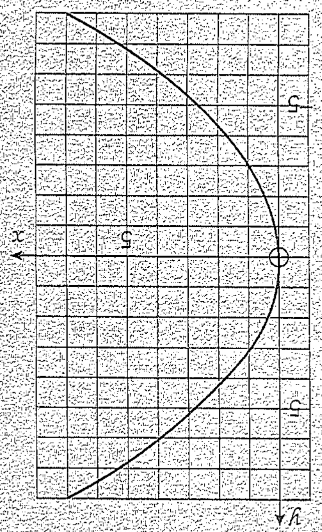

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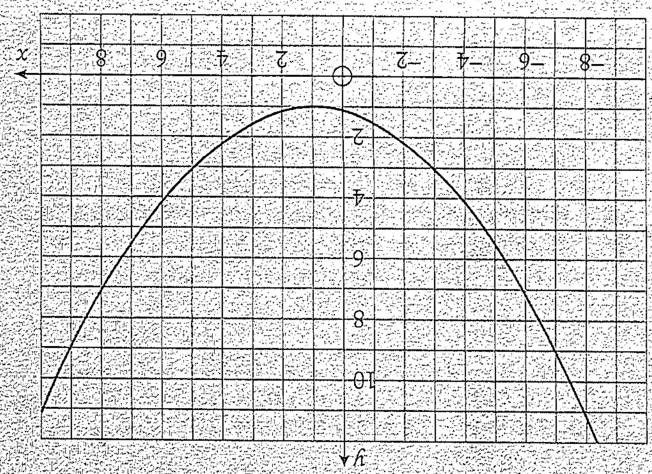


Exercise 3.5C A3

1 Vertex (0, 0) and focus (3, 0) 2 Vertex (0, 0) and focus (6.25, 0)

3 Vertex ($^-3$, 2) and focus ($^-1$, 2) 4 Vertex (0, 1) and focus (3, 1)

5 Vertex (0, 0) and focus (1.5, 0) 6 Vertex (1, $^-2$) and focus (1, 1)

7 Equation: $y^2 = 8(x-1)$ 8 Equation: $(y-1)^2 = 4(x+2)$

9 Equation: $y^2 = 9x$ 10 Equation: $(x-1)^2 = 8(y-1)$

Exercise 3.5C A4 on CD

Sketch the following hyperbolae. (Remember to show coordinates of the centre, axis intercepts and asymptotes clearly.)

1 $\frac{x^2}{9} - \frac{y^2}{25} = 1$

2 $x^2 - \frac{y^2}{16} = 1$

3 $\frac{(x-3)^2}{4} - \frac{(y+1)^2}{16} = 1$

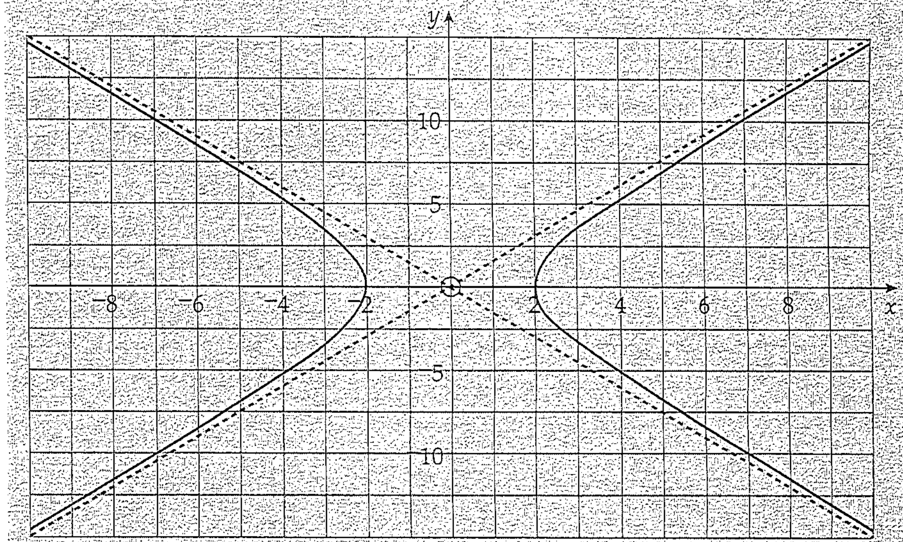
4 $\frac{(x-4)^2}{9} - \frac{(y+2)^2}{16} = 1$

5 $x^2 + 6x - 4y^2 + 16y - 43 = 0$

6 $x^2 - 4y^2 + 2x + 24y - 135 = 0$

Give the equation of the following hyperbolae.

7



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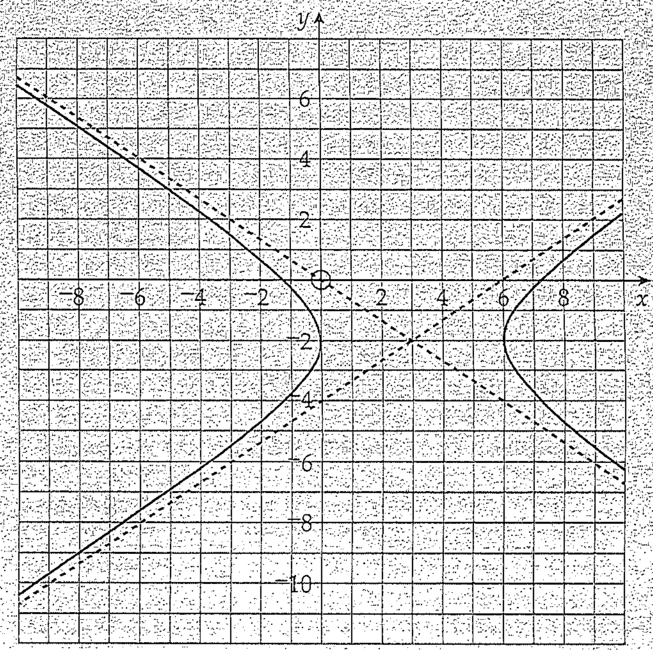


Exercise 3.5C A4

1 Centre (0, 0), equations of asymptotes: $y = \pm\frac{5}{3}x$

2 Centre (0, 0), equations of asymptotes: $y = \pm 4x$

3 Centre (3, −1), equations of asymptotes: $(y + 1) = \pm 2(x - 3)$

4 Centre (4, −2), equations of asymptotes: $(y + 2) = \pm\frac{4}{3}(x - 4)$

5 Centre (−3, 2), equations of asymptotes: $(y - 2) = \pm\frac{1}{2}(x + 3)$

6 Centre (−1, 3), equations of asymptotes: $(y - 3) = \pm\frac{1}{2}(x + 1)$

7 Equation of hyperbola: $\frac{x^2}{4} - \frac{y^2}{9} = 1$

8 Equation of hyperbola: $\frac{(x-3)^2}{9} - \frac{(y+2)^2}{4} = 1$

Solutions

Exercise 3.5C A5

1 Parametric equations: $x = 3\cos(\theta)$, $y = 3\sin(\theta)$

2 Parametric equations: $x = 4\cos(\theta)$, $y = 4\sin(\theta) + 3$

3 Parametric equations: $x = 3t^2$, $y = 6t$

4 Parametric equations: $x = t^2 + 1$, $y = 2t + 1$

5 Parametric equations: $x = 9\cos(\theta)$, $y = 10\sin(\theta)$

6 Parametric equations: $4\sec(\theta) + 3$, $y = 5\tan(\theta) - 1$

7 $\dfrac{x^2}{36} + \dfrac{y^2}{25} = 1$

8 $\dfrac{(x+2)^2}{9} + \dfrac{(y-1)^2}{9} = 1$

$(x+2)^2 + (y-1)^2 = 9$

Exercise 3.5C A5 on CD

Find the parametric equations of each of the following:

1 $x^2 + y^2 = 9$

2 $x^2 + (y-3)^2 = 16$

3 $y^2 = 12x$

4 $(y-1)^2 = 4(x-1)$

5 $\dfrac{x^2}{81} + \dfrac{y^2}{100} = 1$

6 $\dfrac{(x-3)^2}{16} - \dfrac{(y+1)^2}{25} = 1$

Eliminate the parameter in the following equations and form a Cartesian equation for a conic section:

7 $x = 6\cos(t)$, $y = 5\sin(t)$

8 $x = 3\cos(t) - 2$, $y = 3\sin(t) + 1$

Exercise 3.5C A5 on CD

Find the parametric equations of each of the following:

1 $x^2 + y^2 = 9$

2 $x^2 + (y-3)^2 = 16$

3 $y^2 = 12x$

4 $(y-1)^2 = 4(x-1)$

5 $\dfrac{x^2}{81} + \dfrac{y^2}{100} = 1$

6 $\dfrac{(x-3)^2}{16} - \dfrac{(y+1)^2}{25} = 1$

Eliminate the parameter in the following equations and form a Cartesian equation for a conic section:

7 $x = 6\cos(t)$, $y = 5\sin(t)$

8 $x = 3\cos(t) - 2$, $y = 3\sin(t) + 1$

Solutions

Exercise 3.5C A5

1 Parametric equations: $x = 3\cos(\theta)$, $y = 3\sin(\theta)$

2 Parametric equations: $x = 4\cos(\theta)$, $y = 4\sin(\theta) + 3$

3 Parametric equations: $x = 3t^2$, $y = 6t$

4 Parametric equations: $x = t^2 + 1$, $y = 2t + 1$

5 Parametric equations: $x = 9\cos(\theta)$, $y = 10\sin(\theta)$

6 Parametric equations: $4\sec(\theta) + 3$, $y = 5\tan(\theta) - 1$

7 $\dfrac{x^2}{36} + \dfrac{y^2}{25} = 1$

8 $\dfrac{(x+2)^2}{9} + \dfrac{(y-1)^2}{9} = 1$

$(x+2)^2 + (y-1)^2 = 9$

Achievement revision

Exercise 3.5C A6 on CD

Sketch the following graphs. Label any intercepts and any asymptotes.

1 $\dfrac{(x-3)^2}{16}+\dfrac{y^2}{9}=1$

2 $4x^2+y^2=64$

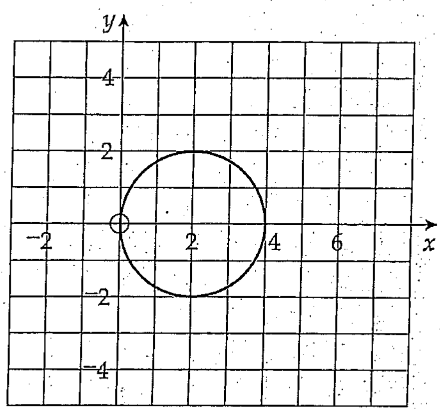
3 $x^2+y^2-10x-12y-20=0$

4 $x=3\cos(\theta)$
$y=3\sin(\theta)+5$

5 $x=3t^2+2$
$y=6t$

Find the equations of the conic sections shown:

6



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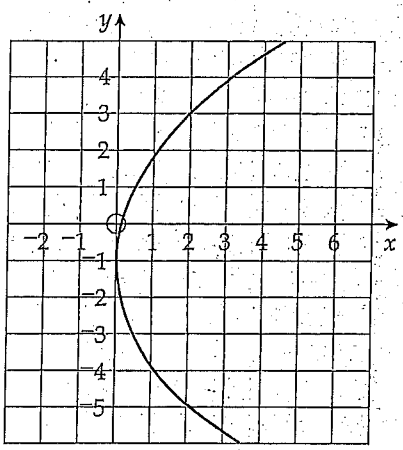

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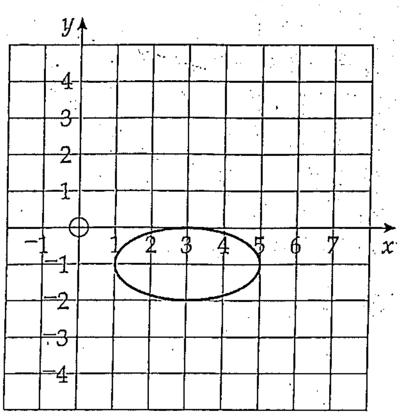

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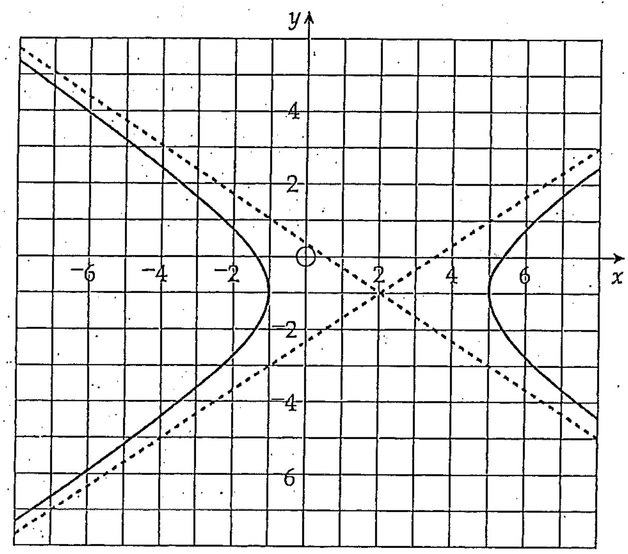

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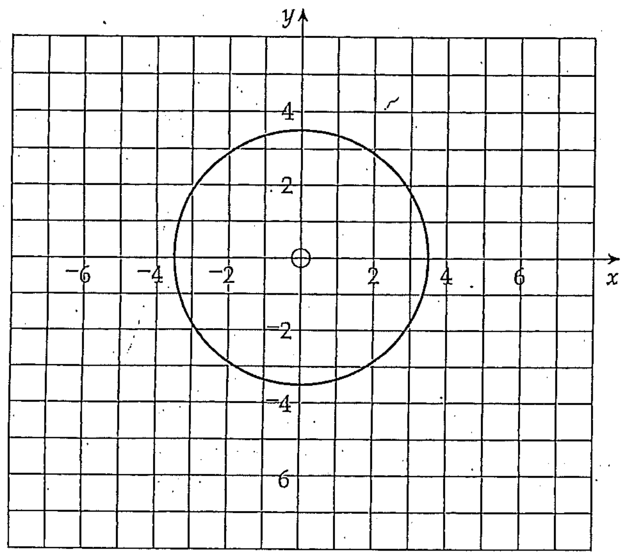


Solutions

Exercise 3.5C A6

1 Ellipse: centre (3, 0), semi-horizontal axis length 4, semi-vertical axis length 3

2 Ellipse: centre (0, 0), semi-horizontal axis length 4, semi-vertical axis length 8

3 Circle: centre (5, 6), radius 9 units

4 Circle: centre (0, 5), radius 3 units

5 Parabola: vertex at (2, 0), focus at (5, 0)

6 $(x-2)^2 + y^2 = 4$

7 $(y+1)^2 = 8x$

8 $\dfrac{(x-3)^2}{4} + (y+1)^2 = 1$

9 $\dfrac{(x-2)^2}{9} - \dfrac{(y+1)^2}{4} = 1$

10 $x^2 + y^2 = 12.25$

Solutions

11 Conic sections – Merit

Exercise 3.5C M1

1 Equation of tangent: $2x + \sqrt{5}y - 9 = 0$

Equation of normal: $2y = \sqrt{5}x$

2 Equation of tangent: $x - 2y + 4 = 0$

Equation of normal: $2x + y - 12 = 0$

3 Equation of tangent: $x - 2y - 72 = 0$

Equation of normal: $2x + y - 29 = 0$

4 Equation of tangent: $y - \sqrt{3} = \frac{2\sqrt{3}}{9}\left(x - \frac{3}{2}\right)$

Equation of normal: $y - \sqrt{3} = \frac{3\sqrt{3}}{2}\left(x - \frac{3}{2}\right)$

Exercise 3.5C M2

1 Points of intersection (4.56, 1.68) and (3.23, $^-$2.31) (3 sf)

2 Points of intersection (7.85, 9.70) and (1.15, $^-$3.70) (3 sf)

3 There is one point of intersection, at $\left(\frac{10}{3}, \frac{-8}{3}\right)$.

As there is only one point of intersection, the line is a tangent to the hyperbola.

Exercise 3.5C M2 on CD

Find the points of intersection between the following conic sections and straight lines:

1 The ellipse $x^2 + 4y^2 = 32$ and the line $y = 3x - 12$.

2 The parabola $y^2 = 12x$ and the line $y = 2x - 6$.

3 The hyperbola $x^2 - y^2 = 4$ and the line $y = x - 6$.

Explain the significance of your answer to this question.

Exercise 3.5C M1 on CD

Find the equations of the tangents and normals at the given points in the following conics:

1 The circle $x^2 + y^2 = 9$ at the point $(2, \sqrt{5})$.

2 The parabola $y^2 = 4x$ at the point (4, 4).

3 The parabola $x = 6t^2 + 2$ and $y = 1 - 12t$ at the point when $t = 2$.

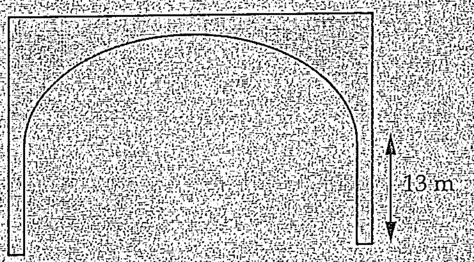
4 The ellipse $x = 3\cos(t)$ and $y = 2\sin(t)$ at the point $\left(\frac{3}{2}, \sqrt{3}\right)$.

Exercise 3.5C M3 on CD

1 Find the equation of the tangent to the ellipse $\frac{x^2}{81} + \frac{y^2}{9} = 1$ at the point $(3, \sqrt{8})$.

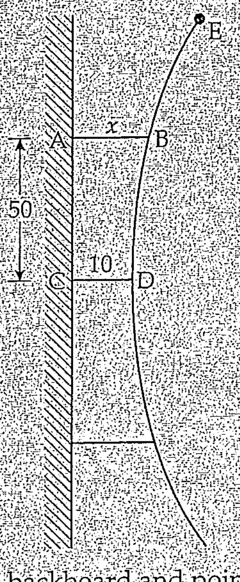
2 A curve is given by the equations $x = 4\cos(\theta) + 1$ and $y = 4\sin(\theta)$. Find the equation of the normal to this curve at the point where $\theta = \frac{\pi}{4}$.

3 The diagram opposite shows the cross-section of a portion of the Icelandic Keflavik Harbour Bridge.



The curved overhead portion is elliptical in shape. Letting the axis of symmetry be the y-axis and the water level be the x-axis, find the equation of the elliptical part of the bridge if the highest point is 22 metres above the water, the total width of the arch is 30 m and the distance from the water to the start of the elliptical curve is 13 m.

4 The shape of a baseball pitcher's screen can be modelled by one side of a hyperbola, as shown in the diagram.



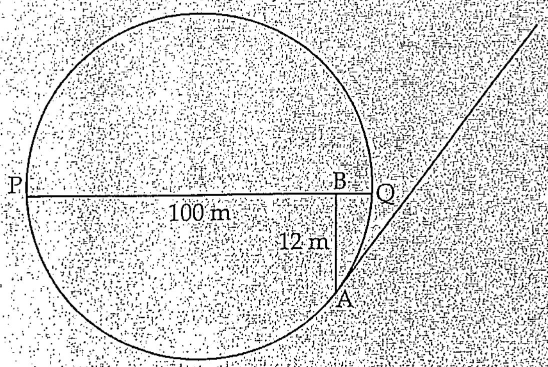
The line AC is on the backboard and points B, D and E are all on the hyperbola-shaped screen. The support dowel CD is 10 cm long and the point E is a vertical distance of 180 cm above D, and 200.25 cm from the backboard. How long is the support dowel AB if the distance AC is 50 cm?

5 Braithwaite School has a practice running track which is circular and has a diameter of 100 m. This means the students cannot

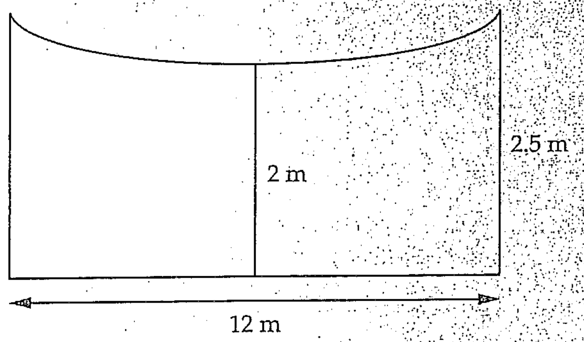
MERIT REVISION

easily practise their times for 400 m races. To solve the problem, the groundskeeper adds a straight length of track which runs at a tangent from the circular track.

The track leaves the circular track at A, which is a distance of 12 m from B which lies on the diameter (PQ) of the circular track. Find the distance QB.



6 At the Scout Jamboree in Napier, the patrols are setting up washing lines. Each patrol is provided with two 2.8-metre length poles, one 2.3-metre support pole and a length of washing line. The scouts are given a diagram to help them and told that they need to set the outside poles 12 metres apart and to dig the poles



30 cm into the ground. The resulting washing line should be able to be modelled as a parabola.

Peter has a maximum reach of 2.2 m. What is the greatest distance he can be from the centre pole and still be able to just touch the washing line?

Exercise 3.5C M3

1 Equation of tangent: $x + 3\sqrt{8}y - 27 = 0$

2 $y = x - 1$

3 Equation of ellipse: $\frac{x^2}{225} + \frac{(y-13)^2}{81} = 1$

4 $x = 56.45$ cm

5 Length of QB is 1.46 units.

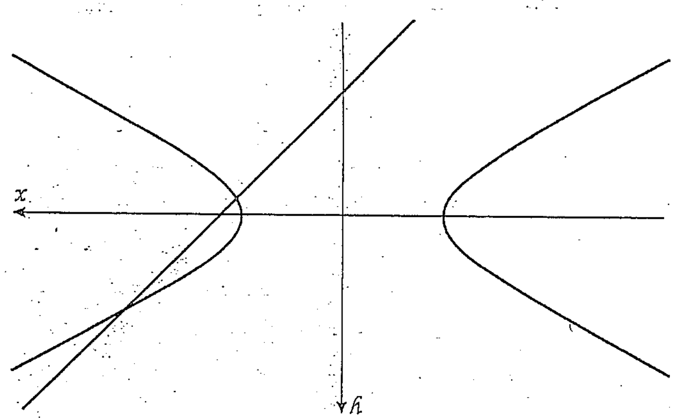
6 Peter will only be able to reach the line for up to 3.79 (3 sf) metres from the centre.

EXCELLENCE REVISION

Exercise 3.5C E1

For all these questions you will be expected to show all working and set out your solutions in clear, logical order.

1 The hyperbola $x^2 - 4y^2 = 6$ is intersected twice by the straight line $ax + by + c = 0$. Find the coordinates of the points of intersection. (Give your answers in terms of a, b and c.)



2 Show that the locus of the vertex of the parabola $y = x^2 + px + q$ is itself a parabola, and find the equation of this parabola in terms of p and q.

3 Like the other solar system planets, Saturn's orbit is elliptical with the Sun at one focus. When it is closest to the Sun (at *perihelion*), Saturn lies 9.03 Astronomical Units (AU) from the Sun; when furthest away (*aphelion*), it is 10.06 AU from the Sun. Using AU, find the equation which would closely model the orbit of Saturn around the Sun.

4 If $y = mx + c$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, show that $a^2 = \frac{b^2 + c^2}{m^2}$.

5 Kate wants to cut out an elliptical shape from a piece of card measuring 48 cm by 15 cm. She knows that she can draw the ellipse by placing a pin in each focal point and attaching a piece of string which is twice the length of the ellipse between them. Where must Kate put the pins and how long must the string be for the ellipse to touch all four sides of the cardboard?

12 Conic sections — Excellence

Exercise 3.4C E1

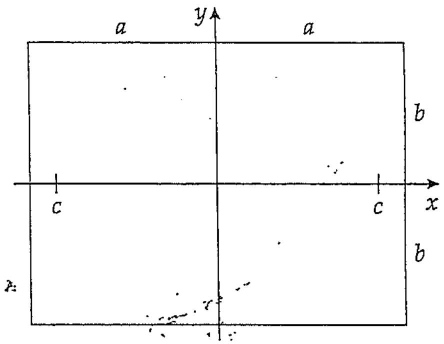
1 $x = \frac{4ac \pm \sqrt{2b^2(3b^2 - 2c^2 - 12a^2)}}{b^2 - 4a^2}$

2 The vertex is at: $\left(\frac{-p}{2}, -\left(q - \frac{p^2}{4}\right)\right)$

3 Equation of orbit: $\frac{x^2}{91.11} - \frac{y^2}{90.84} = 1$

4 $a^2 = \frac{b^2 + c^2}{m^2}$

5



Pins at (22.8, 0) and (−22.8, 0). String must be $2a = 48$ cm long.